

五年級抗疫加油站練習 2

分數裂項初步

1) 試計算 $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{99 \times 100}$ 。

2) 試計算 $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots + \frac{1}{2450}$ 。

3) 試計算 $\frac{2}{1 \times 3} + \frac{2}{3 \times 5} + \frac{2}{5 \times 7} + \frac{2}{7 \times 9} + \dots + \frac{2}{47 \times 49}$ 。

1.

【答案】 $\frac{99}{100}$

【解析】
$$\begin{aligned} & \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{99 \times 100} \\ &= \frac{2-1}{1 \times 2} + \frac{3-2}{2 \times 3} + \frac{4-3}{3 \times 4} + \frac{5-4}{4 \times 5} + \dots + \frac{100-99}{99 \times 100} \\ &= \frac{2}{1 \times 2} - \frac{1}{1 \times 2} + \frac{3}{2 \times 3} - \frac{2}{2 \times 3} + \frac{4}{3 \times 4} - \frac{3}{3 \times 4} + \frac{5}{4 \times 5} - \frac{4}{4 \times 5} + \dots + \frac{100}{99 \times 100} - \frac{99}{99 \times 100} \\ &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{99} - \frac{1}{100} \\ &= 1 - \frac{1}{100} \\ &= \frac{99}{100} \end{aligned}$$

2.

【答案】 $\frac{12}{25}$

【解析】
$$\begin{aligned} & \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots + \frac{1}{2450} \\ &= \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \dots + \frac{1}{49 \times 50} \\ &= \frac{1}{2} - \frac{1}{50} \\ &= \frac{12}{25} \end{aligned}$$

3.

【答案】 $\frac{48}{49}$

【解析】
$$\begin{aligned} & \frac{2}{1 \times 3} + \frac{2}{3 \times 5} + \frac{2}{5 \times 7} + \frac{2}{7 \times 9} + \dots + \frac{2}{47 \times 49} \\ &= \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \dots + \frac{1}{47} - \frac{1}{49} \\ &= 1 - \frac{1}{49} \\ &= \frac{48}{49} \end{aligned}$$